

GROUPS OF ORDERS 1-15

SUMMARY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
# abelian	1	1	1	2	1	1	1	3	2	1	1	2	1	1	1
# other decomposable	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0
#other indecomposable	0	0	0	0	0	1	0	2	0	0	0	2	0	0	0
TOTAL	1	1	1	2	1	2	1	5	2	2	1	5	1	2	1

$$1 = \langle \rangle$$

elts	1
χ_1	1
order	1

The group of order 1 is ubiquitous in that it occurs as a subgroup and a quotient group of every group. Its existence is the simplest way of showing that the group axioms are consistent. We denote the trivial group by 1, and its only element is also denoted by 1. That element has order 1 and the only entry in its character table is the number 1. It is the first cyclic group and also the symmetric group S_1 .

$$2 = \langle A^2 \rangle$$

elts	1	A
χ_1	1	1
χ_2	1	-1
order	1	2

The cyclic group of order 2 is also the symmetric group S_2 . It is also the dihedral group D_2 . It is the only non-trivial group of integers under multiplication.

$$3 = \langle A^3 \rangle$$

elts	1	A	A ²
χ_1	1	1	1
χ_2	1	ω	ω^2
χ_3	1	ω^2	ω
order	1	3	3

The cyclic group of order 3 is the rotation group of the insignia of the Isle of Man.



$$4 = \langle A^4 \rangle$$

elts	1	A ²	A	A ³	
CC	1	2	3	4	\mathfrak{K}
χ_1	1	1	1	1	G
χ_2	1	1	-1	-1	1
χ_3	1	-1	i	-i	0
χ_4	1	-1	-i	i	0
order	1	2	4	4	

When we come to order 4, we get two different groups. The first is cyclic. In fact there is always a cyclic group of any order.

$$4.2 = \langle A^2, B^2 \rangle = D_4 = 2^2$$

class	size	order	class	size	order
1 = 1 × 1	1	1	4 = 1 × 2	1	2
2 = 2 × 1	1	2	5 = 2 × 2	1	2

CC	1	2	3	4	\mathfrak{K}
χ_1	1	1	1	1	G
χ_2	1	1	-1	-1	1
χ_3	1	-1	1	-1	2
χ_4	1	-1	-1	1	3
order	1	2	2	2	

Normal subgroups

Classes	H	G/H	\cap
1 + 2	2	2	1
1 + 3	2	2	2
1 + 4	2	2	3

$$5 = \langle A^5 \rangle$$

elts	1	A	A ⁴	A ²	A ³	
χ_1	1	1	1	1	1	
χ_2	1	θ	θ^4	θ^2	θ^3	
χ_3	1	θ^4	θ	θ^3	θ^2	
χ_4	1	θ^2	θ^3	θ^4	θ	
χ_5	1	θ^3	θ^2	θ	θ^4	$\theta = e^{2\pi i/5}$
order	1	5	5	5	5	

$$6 = \langle A^6 \rangle = 3 \times 2$$

elts	1	A ²	A ⁴	A ³	A ⁵	A	
CC	1	2	3	4	5	6	\mathcal{K}
χ_1	1	1	1	1	1	1	G
χ_2	1	ω	ω^2	1	ω	ω^2	1
χ_3	1	ω^2	ω	1	ω^2	ω	1
χ_4	1	1	1	-1	-1	-1	2
χ_5	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	0
χ_6	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	0
order	1	3	3	2	6	6	

The group 6.1 is the smallest decomposable cyclic group.

Normal Subgroups

Classes	H	G/H	\cap	
1	1 + 4	2	3	1
2	1 + 2 + 3	3	2	2

Sylow and Maximal subgroups: [2], [3]

$$6.2 = D_6 = S_3 = \langle A^3, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3
elts	1	A A ²	B AB A ² B

CC	1	2	3	
size	1	2	3	\mathcal{K}
χ_1	1	1	1	G
χ_2	1	1	-1	1
χ_3	2	-1	0	1
order	1	3	2	

As well as being the smallest non-abelian symmetric group, 6.2 is the smallest non-abelian dihedral group, being the group of rotations of an equilateral triangle.

Normal Subgroups

Classes	H	G/H	Z = 1	
1	1 + 2	3	2	G' = Φ

$$Z = N = \Phi = [1]$$

Sylow and Maximal Subgroups: [2] \times 3, [3]

$$7 = \langle A^7 \rangle$$

$$8 = \langle A^8 \rangle$$

elts	1	A ⁴	A ²	A ⁶	A	A ⁷	A ³	A ⁵	
CC	1	2	3	4	5	6	7	8	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	-1	-1	-1	1
χ_3	1	1	-1	-1	i	-i	-i	i	2
χ_4	1	1	-1	-1	-i	i	i	-i	2
χ_5	1	-1	i	-i	θ	θ^7	θ^3	θ^5	0
χ_6	1	-1	-i	i	θ^7	θ	θ^5	θ^3	0
χ_7	1	-1	-i	i	θ^3	θ^5	θ	θ^7	0
χ_8	1	-1	i	-i	θ^5	θ^3	θ^7	θ	0
order	1	2	4	4	8	8	8	8	

$\theta = e^{2\pi i/8}$

Normal Subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4	4	2	1
2	1 + 2	2	4	2

$\mathfrak{K} = \Phi$

Maximal subgroups: [4]

$$8.2 = 4 \times 2 = \langle A^4, B^2 \rangle$$

class	size	order	class	size	order
1 = 1 × 1	1	1	5 = 1 × 2	1	2
2 = 2 × 1	1	2	6 = 2 × 2	1	2
3 = 3 × 1	1	4	7 = 3 × 2	1	4
4 = 4 × 1	1	4	8 = 4 × 2	1	4

CC	1	2	3	4	5	6	7	8	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	-1	1	1	-1	-1	1
χ_3	1	-1	i	-i	1	-1	i	-i	2
χ_4	1	-1	-i	i	1	-1	-i	i	2
χ_5	1	1	1	1	-1	-1	-1	-1	3
χ_6	1	1	-1	-1	-1	-1	1	1	4
χ_7	1	-1	i	-i	-1	1	-i	i	5
χ_8	1	-1	-i	i	-1	1	i	-i	5
order	1	2	4	4	2	2	4	4	

Normal Subgroups

Classes	H	G/H	\cap	Classes	H	G/H	\cap		
1	1 + 2 + 5 + 6	2 ²	2	1	5	1 + 6	2	4	5
2	1 + 5	2	4	2	6	1 + 2	2	2 ²	1 ∩ 3
3	1 + 2 + 3 + 4	4	2	3					$\mathfrak{K} = \Phi$
4	1 + 2 + 7 + 8	2 ²	2	4					

Maximal subgroups: [2²] × 2, [4]

$$8.3 = \langle A^2, B^2, C^2 \rangle = 2^3 = 2^2 \times 2$$

Class	size	order	class	size	order
1 = 1 × 1	1	1	5 = 1 × 2	1	2
2 = 2 × 1	1	2	6 = 2 × 2	1	2
3 = 3 × 1	1	2	7 = 3 × 2	1	2
4 = 4 × 1	1	2	8 = 4 × 2	1	2

CC	1	2	3	4	5	6	7	8	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	-1	1	1	-1	-1	1
χ_3	1	-1	1	-1	1	-1	1	-1	2
χ_4	1	-1	-1	1	1	-1	-1	1	3
χ_5	1	1	1	1	-1	-1	-1	-1	4
χ_6	1	1	-1	-1	-1	-1	1	1	5
χ_7	1	-1	1	-1	-1	1	-1	1	6
χ_8	1	-1	-1	1	-1	1	1	-1	7
order	1	2	2	2	2	2	2	2	

Normal Subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 5 + 6	2^2	2	1
2	1 + 3 + 5 + 7	2^2	2	2
3	1 + 4 + 5 + 8	2^2	2	3
4	1 + 2 + 3 + 4	2^2	2	4
5	1 + 2 + 7 + 8	2^2	2	5
6	1 + 3 + 6 + 8	2^2	2	6
7	1 + 4 + 6 + 7	2^2	2	7
8	1 + 5	2	2^2	$1 \cap 2$
8	1 + 2	2	2^2	$1 \cap 4$
9	1 + 6	2	2^2	$1 \cap 6$
10	1 + 3	2	2^2	$2 \cap 4$
11	1 + 7	2	2^2	$2 \cap 5$
12	1 + 4	2	2^2	$3 \cap 4$
13	1 + 8	2	2^2	$3 \cap 5$

Maximal subgroups: $[2^2] \times 7$

$$8.4 = D_8 = \langle A^4, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5
elts	1	A ²	A A ³	B A ² B	AB A ³ B

CC	1	2	3	4	5	
Size	1	1	2	2	2	\mathcal{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	1	-1	-1	1
χ_3	1	1	-1	1	-1	2
χ_4	1	1	-1	-1	1	3
χ_5	2	-2	0	0	0	0
order	1	2	4	2	2	

The group 8.4 is the dihedral group of order 8, being the rotation group of a square.

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3	4	2	1
2	1 + 2 + 4	2 ²	2	2
3	1 + 2 + 5	2 ²	2	3
4	1 + 2	2	2 ²	1 \cap 2

$G' = Z = \Phi = \mathcal{K}$

Maximal subgroups: [4], [2²] × 2

$$8.5 = Q_8 = \langle A^4, B^2 = A^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5
elts	1	A ²	A A ³	B A ² B	AB A ³ B

CC	1	2	3	4	5	
size	1	1	2	2	2	\mathcal{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	1	-1	-1	1
χ_3	1	1	-1	1	-1	2
χ_4	1	1	-1	-1	1	3
χ_5	2	-2	0	0	0	1
order	1	2	4	4	4	

Q_8 is Hamiltonian (every subgroup is normal)

The group Q_8 is the first of the quaternion groups. Note that D_8 and Q_8 have the same character tables. They differ only in the orders of the elements. Q_8 was discovered by Hamilton, where he described it as: $\{\pm 1, \pm i, \pm j, \pm k\}$ with

$$i^2 = j^2 = k^2 = -1 \text{ and } ij = k, jk = i, ki = j.$$

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3	4	2	1
2	1 + 2 + 4	4	2	2
3	1 + 2 + 5	4	2	3
4	1 + 2	2	2 ²	1 \cap 2

$G' = Z = \Phi$

Maximal subgroups: [4] × 3

$$9 = \langle A^9 \rangle \quad \theta = e^{2\pi i/9}$$

elts	1	A ³	A ⁶	A	A ⁸	A ²	A ⁷	A ⁴	A ⁵	
CC	1	2	3	4	5	6	7	8	9	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	1	ω	ω^2	ω^2	ω	ω	ω^2	1
χ_3	1	1	1	ω^2	ω	ω	ω^2	ω^2	ω	1
χ_4	1	ω	ω^2	θ	$\omega^2\theta^2$	θ^2	$\omega^2\theta$	$\omega\theta$	$\omega\theta^2$	0
χ_5	1	ω^2	ω	$\omega^2\theta^2$	θ	$\omega^2\theta$	θ^2	$\omega\theta^2$	$\omega\theta$	0
χ_6	1	ω^2	ω	θ^2	$\omega^2\theta$	$\omega\theta$	$\omega\theta^2$	θ	$\omega^2\theta^2$	0
χ_7	1	ω	ω^2	$\omega^2\theta$	θ^2	$\omega\theta^2$	$\omega\theta$	$\omega^2\theta^2$	θ	0
χ_8	1	ω	ω^2	$\omega\theta$	$\omega\theta^2$	θ	$\omega^2\theta^2$	$\omega^2\theta$	θ^2	0
χ_9	1	ω^2	ω	$\omega\theta^2$	$\omega\theta$	$\omega^2\theta^2$	θ	θ^2	$\omega^2\theta$	0
order	1	3	3	9	9	9	9	9	9	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 2 + 3	3	3

Φ Maximal subgroups: [3]

$$9.2 = \langle A^3, B^3 \rangle = 3^2$$

class	Size	order	Class	size	order	class	size	order
1 = 1 × 1	1	1	4 = 1 × 2	1	3	7 = 1 × 3	1	3
2 = 2 × 1	1	3	5 = 2 × 2	1	3	8 = 2 × 3	1	3
3 = 3 × 1	1	3	6 = 3 × 2	1	3	9 = 3 × 3	1	3

CC	1	2	3	4	5	6	7	8	9	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	G
χ_2	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1
χ_3	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1
χ_4	1	1	1	ω	ω	ω	ω^2	ω^2	ω^2	2
χ_5	1	ω	ω^2	ω	ω^2	1	ω^2	1	ω	3
χ_6	1	ω^2	ω	ω	1	ω^2	ω^2	ω	1	4
χ_7	1	1	1	ω^2	ω^2	ω^2	ω	ω	ω	2
χ_8	1	ω	ω^2	ω^2	1	ω	ω	ω^2	1	4
χ_9	1	ω^2	ω	ω^2	ω	1	ω	1	ω^2	3
order	1	3	3	3	3	3	3	3	3	

Normal subgroups

Classes	H	G/H	\cap
1	1 + 4 + 7	3	3
2	1 + 2 + 3	3	3
3	1 + 6 + 8	3	3
4	1 + 5 + 9	3	3

Maximal subgroups: [3] × 4

$$10 = \langle A^{10} \rangle = 5 \times 2 \quad \theta = e^{2\pi i/5}$$

class	size	order	class	size	Order
1 = 1 × 1	1	1	6 = 1 × 2	1	2
2 = 2 × 1	1	5	7 = 2 × 2	1	10
3 = 3 × 1	1	5	8 = 3 × 2	1	10
4 = 4 × 1	1	5	9 = 4 × 2	1	10
5 = 5 × 1	1	5	10 = 5 × 2	1	10

CC	1	2	3	4	5	6	7	8	9	10	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	θ	θ^2	θ^3	θ^4	1	θ	θ^2	θ^3	θ^4	1
χ_3	1	θ^2	θ^4	θ	θ^3	1	θ^2	θ^4	θ	θ^3	1
χ_4	1	θ^3	θ	θ^4	θ^2	1	θ^3	θ	θ^4	θ^2	1
χ_5	1	θ^4	θ^3	θ^2	θ	1	θ^4	θ^3	θ^2	θ	1
χ_6	1	1	1	1	1	-1	-1	-1	-1	-1	2
χ_7	1	θ	θ^2	θ^3	θ^4	-1	$-\theta$	$-\theta^2$	$-\theta^3$	$-\theta^4$	0
χ_8	1	θ^2	θ^4	θ	θ^3	-1	$-\theta^2$	$-\theta^4$	$-\theta$	$-\theta^3$	0
χ_9	1	θ^3	θ	θ^4	θ^2	-1	$-\theta^3$	$-\theta$	$-\theta^4$	$-\theta^2$	0
χ_{10}	1	θ^4	θ^3	θ^2	θ	-1	$-\theta^4$	$-\theta^3$	$-\theta^2$	$-\theta$	0
order	1	5	5	5	5	10	10	2	10	10	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 6	2	5	1
2	1 + 2 + 3 + 4 + 5	5	2	2

Sylow and maximal subgroups: [2], [5]

$$10.2 = \langle A^5, B^2, BA = A^{-1}B \rangle = D_{10}$$

CC	1	2	3	4
elts	1	A A ⁴	A ² A ³	B AB A ² B A ³ B A ⁴ B

CC	1	2	3	4	\mathfrak{K}
size	1	2	2	5	
χ_1	1	1	1	1	G
χ_2	1	1	1	-1	1
χ_3	2	c_1	c_2	0	0
χ_4	2	c_2	c_1	0	0
order	1	5	5	2	

The group D_{10} is the rotation group of a regular pentagon.
 $c_k = 2\cos(2k\pi/5)$

Normal subgroups

Classes	H	G/H		
1	1 + 2 + 3	5	2	G'

$$Z = \mathfrak{Z} = \Phi = [1]$$

Sylow and maximal subgroups: [2] × 5, [5]

$$11 = \langle A^{11} \rangle$$

$$12 = \langle A^{12} \rangle = 4 \times 3$$

class	size	order	class	size	order	class	size	order
1 = 1 × 1	1	1	5 = 1 × 2	1	3	9 = 1 × 3	1	3
2 = 2 × 1	1	2	6 = 2 × 2	1	6	10 = 2 × 3	1	6
3 = 3 × 1	1	4	7 = 3 × 2	1	12	11 = 3 × 3	1	12
4 = 4 × 1	1	4	8 = 4 × 2	1	12	12 = 4 × 3	1	12

CC	1	2	3	4	5	6	7	8	9	10	11	12	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
χ_3	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	2
χ_4	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	2
χ_5	1	1	1	1	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	3
χ_6	1	1	-1	-1	ω	ω	$-\omega$	$-\omega$	ω^2	ω^2	$-\omega^2$	$-\omega^2$	4
χ_7	1	-1	i	-i	ω	$-\omega$	$i\omega$	$-i\omega$	ω^2	$-\omega^2$	$i\omega^2$	$-i\omega^2$	0
χ_8	1	-1	-i	i	ω	$-\omega$	$-i\omega$	$i\omega$	ω^2	$-\omega^2$	$-i\omega^2$	$i\omega^2$	0
χ_9	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	3
χ_{10}	1	1	-1	-1	ω^2	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	$-\omega$	$-\omega$	4
χ_{11}	1	-1	i	-i	ω^2	$-\omega^2$	$i\omega^2$	$-i\omega^2$	ω	$-\omega$	$i\omega$	$-i\omega$	0
χ_{12}	1	-1	-i	i	ω^2	$-\omega^2$	$-i\omega^2$	$i\omega^2$	ω	$-\omega$	$-i\omega$	$i\omega$	0
order	1	4	2	4	3	12	6	12	3	12	6	12	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 5 + 6 + 9 + 10	6	2	1
2	1 + 5 + 9	3	4	2
3	1 + 2 + 3 + 4	4	3	3
4	1 + 2	2	6	4 Φ

Sylow subgroups: [3], [4] Maximal subgroups: [4], [6]

$$12.2 = \langle A^6, B^2 \rangle = 6 \times 2$$

class	size	order	Class	size	order
1 = 1 × 1	1	1	7 = 1 × 2	1	2
2 = 2 × 1	1	3	8 = 2 × 2	1	6
3 = 3 × 1	1	3	9 = 3 × 2	1	6
4 = 4 × 1	1	6	10 = 4 × 2	1	6
5 = 5 × 1	1	2	11 = 5 × 2	1	2
6 = 6 × 1	1	6	12 = 6 × 2	1	6

CC	1	2	3	4	5	6	7	8	9	10	11	12	\mathcal{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1
χ_3	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1
χ_4	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	2
χ_5	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	3
χ_6	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	3
χ_7	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	4
χ_8	1	ω	ω^2	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	-1	$-\omega$	$-\omega^2$	5
χ_9	1	ω^2	ω	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	-1	$-\omega^2$	$-\omega$	5
χ_{10}	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	6
χ_{11}	1	ω	ω^2	-1	$-\omega$	$-\omega^2$	-1	$-\omega$	$-\omega^2$	1	ω	ω^2	7
χ_{12}	1	ω^2	ω	-1	$-\omega^2$	$-\omega$	-1	$-\omega^2$	$-\omega$	1	ω^2	ω	7
order	1	3	3	2	6	6	2	6	6	2	6	6	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 4 + 7 + 10	2^2	3	1
2	1 + 2 + 3 + 7 + 8 + 9	6	2	2
3	1 + 7	2	6	3
4	1 + 2 + 3 + 4 + 5 + 6	6	2	4
5	1 + 4	2	6	5
6	1 + 2 + 3 + 10 + 11 + 12	6	2	6
7	1 + 10	2	6	7
8	1 + 2 + 3	3	2^2	$2 \cap 4$

Sylow subgroups: $[2^2]$, $[3]$ Maximal subgroups: $[2^2]$, $[6]$

$$12.3 = \langle A^3, B^2, C^2, BA = A^{-1}B \rangle = D_{12} = D_6 \times 2$$

CC	1	2	3	4	5	6	
Size	1	2	3	1	2	3	\mathfrak{K}
χ_1	1	1	1	1	1	1	G
χ_2	1	1	-1	1	1	-1	1
χ_3	2	-1	0	2	-1	0	2
χ_4	1	1	1	-1	-1	-1	3
χ_5	1	1	-1	-1	-1	1	4
χ_6	2	-1	0	-2	1	0	0
order	1	3	2	2	6	2	

Normal subgroups

Classes	H	G/H	\cap	$\Phi = 1$
1	1 + 2 + 4 + 5	6	2	1
2	1 + 4	2	D_6	2 $Z = \mathfrak{Q}$
3	1 + 2 + 3	D_6	2	3
4	1 + 2 + 6	D_6	2	4
5	1 + 2	3	2^2	1 \cap 3 G'

Sylow subgroups: $[2^2] \times 3, [3]$ Maximal subgroups: $[2^2] \times 3, [6], [D_6] \times 2$

$$12.4 = D_{3,4} = \langle A^3, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6
elts	1	B^2	A A^2	$AB^2 A^2B^2$	B AB A^2B	$B^3 AB^3 A^2B^3$

CC	1	2	3	4	5	6	
size	1	1	2	2	3	3	\mathfrak{K}
χ_1	1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	-1	1
χ_3	1	-1	1	-1	i	-i	2
χ_4	1	-1	1	-1	-i	i	2
χ_5	2	2	-1	-1	0	0	3
χ_6	2	-2	-1	1	0	0	0
order	1	2	3	6	4	4	

Normal subgroups

Classes	H	G/H	\cap	
1	1 + 2 + 3 + 4	6	2	1
2	1 + 3	3	4	2 G'
3	1 + 2	2	D_6	3 $Z = \Phi = \mathfrak{Q}$

Sylow subgroups: $[4] \times 3, [3]$ Maximal subgroups: $[4] \times 3, [6]$

$$12.5 = A_4 = \langle A^2, B^2, C^3, CA = ABC, CB = AC \rangle$$

CC	1	2	3	4
elts	1	A B AB	C BC AC ABC	C ² BC ² AC ² ABC ²

CC	1	2	3	4	
size	1	3	4	4	\mathfrak{K}
χ_1	1	1	1	1	G
χ_2	1	1	ω	ω^2	1
χ_3	1	1	ω^2	ω	1
χ_4	3	-1	0	0	0
order	1	2	3	3	

This is the alternating group on 4 symbols.

Normal subgroups

	Classes	H	G/H	
1	1 + 2	2 ²	3	G'

Sylow and maximal subgroups: [2²], [3] × 4

$$13 = \langle A^{13} \rangle$$

$$14 = \langle A^7, B^2 \rangle = 7 \times 2 \quad \theta = e^{2\pi i/7}$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	θ	θ^6	θ^2	θ^5	θ^3	θ^4	1	θ	θ^6	θ^2	θ^5	θ^3	θ^4	1
χ_3	1	θ^6	θ	θ^5	θ^2	θ^4	θ^3	1	θ^6	θ	θ^5	θ^2	θ^4	θ^3	1
χ_4	1	θ^2	θ^5	θ^4	θ^3	θ^6	θ	1	θ^2	θ^5	θ^4	θ^3	θ^6	θ	1
χ_5	1	θ^5	θ^2	θ^3	θ^4	θ	θ^6	1	θ^5	θ^2	θ^3	θ^4	θ	θ^6	1
χ_6	1	θ^3	θ^4	θ^6	θ	θ^2	θ^5	1	θ^3	θ^4	θ^6	θ	θ^2	θ^5	1
χ_7	1	θ^4	θ^3	θ	θ^6	θ^5	θ^2	1	θ^4	θ^3	θ	θ^6	θ^5	θ^2	1
χ_8	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	2
χ_9	1	θ	θ^6	θ^2	θ^5	θ^3	θ^4	-1	$-\theta$	$-\theta^6$	$-\theta^2$	$-\theta^5$	$-\theta^3$	$-\theta^4$	0
χ_{10}	1	θ^6	θ	θ^5	θ^2	θ^4	θ^3	-1	$-\theta^6$	$-\theta$	$-\theta^5$	$-\theta^2$	$-\theta^4$	$-\theta^3$	0
χ_{11}	1	θ^2	θ^5	θ^4	θ^3	θ^6	θ	-1	$-\theta^2$	$-\theta^5$	$-\theta^4$	$-\theta^3$	$-\theta^6$	$-\theta$	0
χ_{12}	1	θ^5	θ^2	θ^3	θ^4	θ	θ^6	-1	$-\theta^5$	$-\theta^2$	$-\theta^3$	$-\theta^4$	$-\theta$	$-\theta^6$	0
χ_{13}	1	θ^3	θ^4	θ^6	θ	θ^2	θ^5	-1	$-\theta^3$	$-\theta^4$	$-\theta^6$	$-\theta$	$-\theta^2$	$-\theta^5$	0
χ_{14}	1	θ^4	θ^3	θ	θ^6	θ^5	θ^2	-1	$-\theta^4$	$-\theta^3$	$-\theta$	$-\theta^6$	$-\theta^5$	$-\theta^2$	0
order	1	7	7	7	7	7	7	2	14	14	14	14	14	14	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 8	2	7	1
2	1 + 2 + 3 + 4 + 5 + 6 + 7	7	2	2

Sylow and maximal subgroups: [2], [7]

$14.2 = D_{14} = \langle A^7, B^2, BA = A^{-1}B \rangle \quad c_k = 2\cos(2k\pi/7)$

CC	1	2	3	4	5						
elts	1	A A ⁶	A ² A ⁵	A ³ A ⁴	B	AB	A ² B	A ³ B	A ⁴ B	A ⁵ B	A ⁶ B

CC	1	2	3	4	5	
size	1	2	2	2	7	\mathfrak{K}
χ_1	1	1	1	1	1	G
χ_2	1	1	1	1	-1	1
χ_3	2	c ₁	c ₂	c ₃	0	0
χ_4	2	c ₃	c ₁	c ₂	0	0
χ_5	2	c ₂	c ₃	c ₁	0	0
order	1	7	7	7	2	

Normal subgroups

	Classes	H	G/H	
1	1 + 2 + 3 + 4	7	2	G'

Sylow and maximal subgroups: [2] × 7, [7]

$15 = \langle A^{15} \rangle = 5 \times 3 \quad \theta = e^{2\pi i/5}$

The smallest cyclic group of composite order where there is no other group of that order.

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	\mathfrak{K}
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ_2	1	θ	θ^4	θ^2	θ^3	1	θ	θ^4	θ^2	θ^3	1	θ	θ^4	θ^2	θ^3	1
χ_3	1	θ^4	θ	θ^3	θ^2	1	θ^4	θ	θ^3	θ^2	1	θ^4	θ	θ^3	θ^2	1
χ_4	1	θ^2	θ^3	θ^4	θ	1	θ^2	θ^3	θ^4	θ	1	θ^2	θ^3	θ^4	θ	1
χ_5	1	θ^3	θ^2	θ	θ^4	1	θ^3	θ^2	θ	θ^4	1	θ^3	θ^2	θ	θ^4	1
χ_6	1	1	1	1	1	ω	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	2
χ_7	1	θ	θ^4	θ^2	θ^3	ω	$\omega\theta$	$\omega\theta^4$	$\omega\theta^2$	$\omega\theta^3$	ω^2	$\omega^2\theta$	$\omega^2\theta^4$	$\omega^2\theta^2$	$\omega^2\theta^3$	0
χ_8	1	θ^4	θ	θ^3	θ^2	ω	$\omega\theta^4$	$\omega\theta$	$\omega\theta^3$	$\omega\theta^2$	ω^2	$\omega^2\theta^4$	$\omega^2\theta$	$\omega^2\theta^3$	$\omega^2\theta^2$	0
χ_9	1	θ^2	θ^3	θ^4	θ	ω	$\omega\theta^2$	$\omega\theta^3$	$\omega\theta^4$	$\omega\theta$	ω^2	$\omega^2\theta^2$	$\omega^2\theta^3$	$\omega^2\theta^4$	$\omega^2\theta$	0
χ_{10}	1	θ^3	θ^2	θ	θ^4	ω	$\omega\theta^3$	$\omega\theta^2$	$\omega\theta$	$\omega\theta^4$	ω^2	$\omega^2\theta^3$	$\omega^2\theta^2$	$\omega^2\theta$	$\omega^2\theta^4$	0
χ_{11}	1	1	1	1	1	ω^2	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	ω	2
χ_{12}	1	θ	θ^4	θ^2	θ^3	ω^2	$\omega^2\theta$	$\omega^2\theta^4$	$\omega^2\theta^2$	$\omega^2\theta^3$	ω	$\omega\theta$	$\omega\theta^4$	$\omega\theta^2$	$\omega\theta^3$	0
χ_{13}	1	θ^4	θ	θ^3	θ^2	ω^2	$\omega^2\theta^4$	$\omega^2\theta$	$\omega^2\theta^3$	$\omega^2\theta^2$	ω	$\omega\theta^4$	$\omega\theta$	$\omega\theta^3$	$\omega\theta^2$	0
χ_{14}	1	θ^2	θ^3	θ^4	θ	ω^2	$\omega^2\theta^2$	$\omega^2\theta^3$	$\omega^2\theta^4$	$\omega^2\theta$	ω	$\omega\theta^2$	$\omega\theta^3$	$\omega\theta^4$	$\omega\theta$	0
χ_{15}	1	θ^3	θ^2	θ	θ^4	ω^2	$\omega^2\theta^3$	$\omega^2\theta^2$	$\omega^2\theta$	$\omega^2\theta^4$	ω	$\omega\theta^3$	$\omega\theta^2$	$\omega\theta$	$\omega\theta^4$	0
order	1	5	5	5	5	3	15	15	15	15	3	15	15	15	15	

Normal subgroups

	Classes	H	G/H	\cap
1	1 + 6 + 11	3	5	1
2	1 + 2 + 3 + 4 + 5	5	3	2

Sylow and maximal subgroups: [3], [5]

